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Crack initiation at weak stress singularities – Finite Fracture Mechanics approach

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Abstract

To study crack initiation at weak stress singularities classical Fracture Mechanics approaches do not suffice. In this work weak singularities are discussed and the obstacles occurring in fracture analysis are outlined. The concept of Finite Fracture Mechanics and the use of coupled criteria is explained. A brief discussion of coupled criteria is given and relationships to other analysis methods are adressed. As an example crack initiation in adhesively bonded joints is analyzed. Two different failure models are outlined and it is shown that both agree well with experimental results.

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Keywords:

1. Introduction

In many engineering applications brittle materials are used. Hence, understanding the failure behaviour of such materials is very important for efficient and save designs. Linear Elastic Fracture Mechanics (LEFM) can be applied to study the behaviour of continua of sufficiently brittle materials that contain cracks. But as we all know structures do not necessarily fail by growth of existing cracks or flaws, cracks can also initiate from local stress concentration. Typical structural situations are holes, notches or multi-material points. An example for the latter case is crack initiation at free edges of laminates. In general there are two different kind of stress concentrations, finite concentrations as e.g. at holes or infinite stress concentrations, typically called stress singularities as they occur in sharp corners or at multi-material points. Several local and non-local concepts have been developed to study such crack initiation processes with the aim to obtain the corresponding crack initiation loads. In this work we will briefly discuss the concept of Finite Fracture Mechanics (FFM) that considers the formation of cracks of finite size and does not require the use of non-physical length parameters. Two FFM failure models for the case of crack initiation in adhesive joints are given and the failure load predictions are compared to experimental results.

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2. Finite Fracture Mechanics

In the vicinity of a crack the stresses show the well-known inverse-square-root stress singularity. The dominant terms of displacements and the stress distribution have the following form:

$$u \sim r^{\frac{1}{2}}; \quad \sigma \sim r^{-\frac{1}{2}}. \tag{1}$$

At geometrical or material disconuities so-called weak stress singularities occur. Typical examples are notches, laminate edges or multi-material points as they occur in bonds. In these cases the exponent of the power-law relation depends on the specific structural situation and changes with geometrical parameters as notch opening angles or with material parameters as the elastic mismatch (Broberg, 1999). The displacements and the stresses close to the disconuities are

$$u \sim r^{\lambda_j}; \quad \sigma \sim r^{-(1-\lambda_j)}. \tag{2}$$

The exponent $1 - \lambda$ is the singularity order. In the case of weak stress singularities it takes values from 0 to 1/2. In certain structural situations even without cracks also strong stress singularities with $1 - \lambda > 1/2$ can occur (Sator and Becker (2012); Goswami and Becker (2012)). Then the singularity order $1 - \lambda$ lies in the range from 1/2 to 1 in 2D and 1/2 to 3/2 in 3D. The energy release rate (ERR) of a crack can be computed by considering the virtual crack closure integral. In this case the crack opening displacements along the crack are multiplied with the stresses along the newly opened crack to obtain the virtual work required to close the crack. This virtual work equals the energy released during crack formation. The strain energy release rate is obtained by letting the crack size approach zero:

$$\mathcal{G}_I = \lim_{\Delta a \to 0} \frac{1}{2\Delta a} \int_0^{\Delta a} \sigma_\perp(r)\Big(u_\perp^+(r) - u_\perp^-(r)\Big)\,\mathrm{d}r \tag{3}$$

In this case the mode I ERR is computed as stresses perpendicular to the crack and the corresponding normal stress opening displacements of the two crack faces (+/-) are considered. If a crack emanating from a point exhibiting a weak stress singularity is considered it can easily be shown that the ERR is zero. The stresses obey relation (2) whereas the most dominating term of the opening behaviour of the newly created crack follows the typical crack opening displacements (1) (Gross and Seelig, 2011). In this case the virtual crack closure integral is

$$\mathcal{G}_I \sim \lim_{\Delta a \to 0} \frac{1}{\Delta a} \int_0^{\Delta a} r^{\lambda-1} r^{\frac{1}{2}}\,\mathrm{d}a \sim \lim_{\Delta a \to 0} \frac{1}{\Delta a} \left[r^{\lambda+\frac{1}{2}}\right]_0^{\Delta a} \overset{\text{l'Hôpital}}{=} \lim_{\Delta a \to 0} \Delta a^{\lambda-\frac{1}{2}} = 0. \tag{4}$$

Thus, classical Fracture Mechanics approaches as Griffith's criterion or the K-concept, e.g.

$$\mathcal{G} = \mathcal{G}_c; \quad K_I = \sqrt{E\mathcal{G}_I} = K_{Ic} \tag{5}$$

fail. It can be concluded that brittle failure at weak stress singularities cannot be covered directly by such approaches. And of course strength of materials approaches cannot be applied directly as well due to the singular nature of the stress field. This problem is overcome in the concept of Finite Fracture Mechanics (FFM) (Hashin, 1996) by considering the instantaneous formation of cracks of finite size. Then the energy condition of Griffith's criterion is considered for the case of finite crack advancement by introducing an incremental energy release rate $\bar{\mathcal{G}}$

$$\bar{\mathcal{G}} = \frac{1}{\Delta A} \int_A^{A+\Delta A} \mathcal{G}(\tilde{A})\mathrm{d}\tilde{A} \tag{6}$$

that is not zero for the case of a weak stress singularity. But of course a new unknown appears in this formulation — the finite crack size ΔA. By introducing a coupled stress and energy criterion Leguillon (2002) has proposed a physical sound model that allows for determination of the unknown critical loading state and the corresponding finite crack size. The coupled criterion requires a simultaneous fulfilment of a Griffith type criterion considering the incremental ERR and of a strength of material criterion:

$$F(\sigma_{ij}(\mathbf{x})) \geq \sigma_c \; \forall \; \mathbf{x} \in \Omega_c \;\; \wedge \;\; \bar{\mathcal{G}}(\Delta A) \geq \mathcal{G}_c. \tag{7}$$

Here, Ω_c is the surface of the considered to-be-initiated crack ΔA. In this coupled criterion only two fundamental failure parameters — the fracture toughness $\mathcal{G}_c$ and the strength σ_c — are required. As an alternative to the point-wise evaluation of the stresses a coupled criterion using an average stress criterion can be formulated (Cornetti et al., 2006):

$$\frac{1}{\Delta A} \int\limits_{\Omega_c} F(\sigma_{ij}(\mathbf{x}))\mathrm{d}\mathbf{x} \geq \sigma_c \;\; \wedge \;\; \bar{\mathcal{G}}(\Delta A) \geq \mathcal{G}_c. \tag{8}$$

The point-wise evaluation of the inequality can in general be given as the condition that the minimal stress on the surface Ω_c must exceed the strength. In the special case of monotonically decaying stresses the condition reverts to an evaluation of the stress at the most distant points from the origin on the crack. The crack area ΔA is not necessarily one connected region. The simultaneous, instantaneous formation of several cracks can be rendered by this approach as well. Such a crack initiation event can occur in the fracturing of a plate with a hole or a center-notch, where two cracks will form (Camanho et al., 2012; Cornetti et al., 2013). Another example is the formation of crack patterns as they occur in thermal or hygro-thermal problems (Leguillon, 2013). In some structural situations with non-singular stress concentrations FFM has shown to give better results than classical strength of materials approaches, especially when size effects occur (Hebel and Becker, 2008; Camanho et al., 2012). In the general case the critical crack initiation loading and the corresponding finite crack size must be found by solving the corresponding restrained optimization problem. The smallest loading must be found that satisfies both criteria of the coupled criterion for any kinematically admissible finite crack configuration:

$$F_f = \min_{F,\Delta A} \left\{ F \mid f(\sigma_{ij}(x_i)) \geq \sigma_c \; \forall x_i \in \Omega_c \;\wedge\; \bar{\mathcal{G}}(\Delta A) \geq \mathcal{G}_c \right\}. \tag{9}$$

The finite crack size in FFM is not arbitrarily chosen but results from the solution of the coupled criterion, which constitutes an advantage over other models that require a length parameter to evaluate failure. To allow for the use of Fracture Mechanics concepts the Imaginary Crack Method makes use of imaginary cracks of length a_0 to study failure of weak stress singularities or non-singular stress concentrations (Waddoups et al., 1971). It is assumed that the crack length is a constant material parameter but it is well known that it changes in different structural situations (Taylor, 2007). Approaches that make use of the Theory of Critical Distances also use an inherent length scale. In these approaches a strength of material approach is followed but the stresses are not evaluated at the local point of the stress concentration. Typical methods are to evaluate the stresses at a certain distance (Point Method), integrated over a certain length (Line Method) or by integrating over an area or volume. This concept was orginally introduced by Neuber who has developed this approach in the 1930s and has refined it the next decades (Neuber, 1961). A very comprehensive overview and good discussion of these approaches can be found in the textbook by Taylor (2007), who also addresses the relationship of these approaches to FFM.

3. Crack initiation at weak stress singularities in adhesive joints

Bonded joints are typically made of two materials with significant elastic mismatch. In case of joints with metallic adherends and epoxy adhesives the ratio of the Young's modulus lies in the range of $0.01 < \frac{E_1}{E_2} < 0.1$. At bi-material interfaces on the surface of adhesive joints stress singularities occur. At rectangular corners of adhesives and adherends as they occur in many joint designs as single lap joints (SLJ) or reinforcement the strongest stress singularities occur. The singularity order attains values of $.2 < 1 - \lambda < .35$ depending on the elastic mismatch and the Poisson's ratios (Bogy, 1971; Weißgraeber and Becker, 2013).

In the following two models of crack initiation in adhesively bonded joints are studied. Fig. 1 shows the considered single lap joint (SLJ) configuration under axial loading F. The thickness of the adherends is denoted by h, the thickness of the adhesive layer with t and L_o is the overlap length. The joint has a constant width b. The adherends and the adhesive are modelled to be homogeneous and isotropic with linear elastic behaviour. The elastic constants of the adherends and the adhesive are E_x, ν_x and E_a, ν_a, respectively.

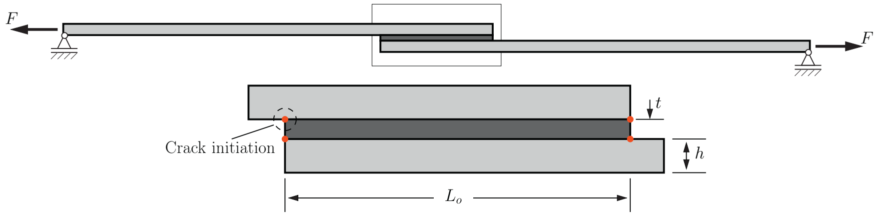

Fig. 1. Considered single lap joint configuration. Weak stress singularities occurring at the bi-material points are marked by a dot. The considered crack initiation point is circled.

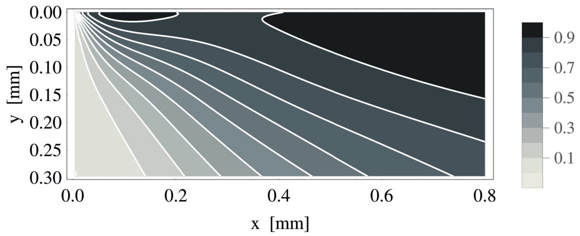

Fig. 2. Incremental energy release rate distribution as a function of the crack tip position. It can be seen that the energy release rate does not increase monotonically with the length of the straight crack.

3.1. Modelling approaches

To study crack initiation in the considered single lap joint configuration two different modelling approaches are employed: a numerical model (Hell et al., 2014) and an analytical model (Weißgraeber and Becker, 2013). In the numerical model a detailed Finite Element analysis is used to identify the stress distribution in the joint and the incremental energy release rates of the crack configuration. Straight cracks in the adhesive layer of length a and angle ϕ to the adherend/adhesive interface are considered. The effect of geometrical non-linearity due to large (bending) deformation of the slender adherends is taken into account by an iterative solution procedure. The incremental energy release rate is obtained by relating the change of potential energy of the cracked and the uncracked state to the finite crack size:

$$\bar{\mathcal{G}}(a,\phi) = -\frac{\Delta\Pi}{ab} = -\frac{\Pi^{(2)}(a,\phi) - \Pi^{(1)}}{ba}. \tag{10}$$

In an automated procedure using Abaqus' python interface the incremental energy release rate of possible crack configurations is obtained. Fig. 2 shows the distribution of the energy release rate as a function of the position of the crack tip as a contour plot. The result shows that the energy release rate does not increase monotonically with the length of the straight crack. This circumstance has to be considered in the optimization problem as it can lead to local minima. Details of the optimization problem can be found in the work by Hell et al. (2014).

In the analytical model the weak interface solution by Ojalvo and Eidinoff (1978) is used to determine the stress distribution in the adhesive layer. The solution models the adherends as beams and the adhesive layer as a simplified continuum (smeared springs). It yields constant peel stresses over the thickness and shear stresses that vary linearly over thickness. As typical for fracture analysis with weak interface models, crack initiation is modelled as shortening of the overlap length (Krenk, 1992), which allows for using the same stress solution in cracked and in uncracked case. Then the differential energy release rate is calculated by considering the energy stored in the smeared springs that represent the adhesive layer at the very end of the overlap:

$$\mathcal{G} = \frac{1}{2G_a}\int_{-t/2}^{t/2} \tau^2 \mathrm{d}z + \frac{1}{2}\frac{t}{E_a}\sigma_{\max}^2 \tag{11}$$

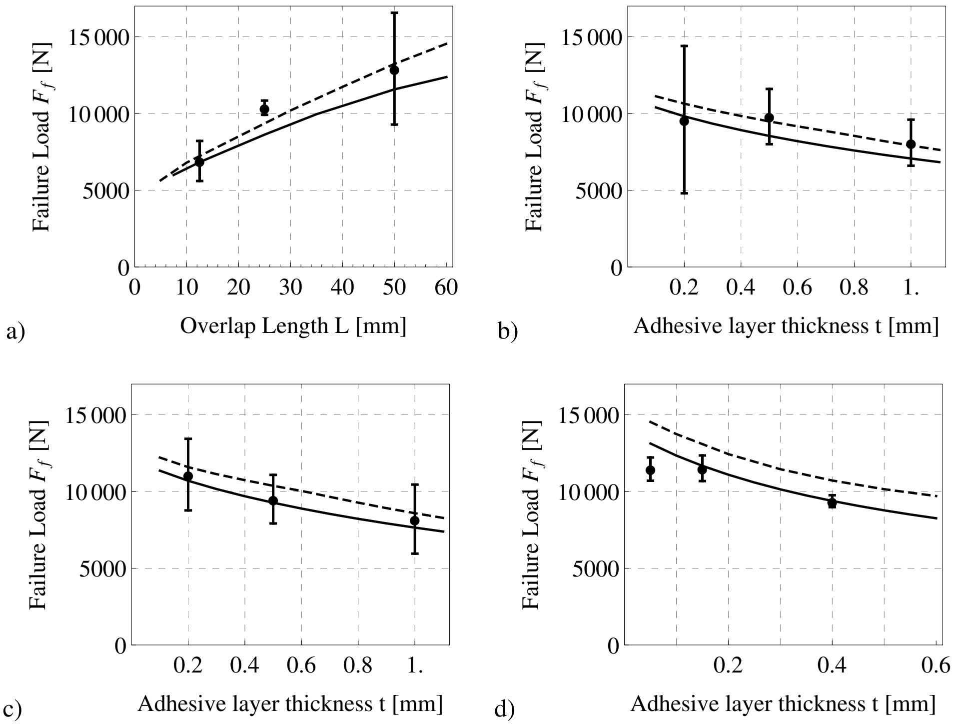

Fig. 3. Comparison of the failure load predictions of the numerical failure model (solid line) and the analytical failure model (dashed line) to experimental results from Castagnetti et al. (2011); da Silva et al. (2004, 2006)

To calculate the incremental energy release rate this result must be integrated over the finite crack length. When crack initiation is modelled as shortening of the overlap length, $dA = -b dL_o$ holds and from (6) we obtain:

$$\bar{\mathcal{G}} = \frac{1}{ba} \int_{L_o - a}^{L_o} \mathcal{G} dL_o \tag{12}$$

This integral has to be evaluated numerically as well as the optimization problem of the coupled criterion.

3.2. Results

The failure load predictions of the outlined failure models of adhesively bonded single lap joints are compared to experimental results to examine the accuracy. Four experimental test series (Castagnetti et al., 2011; da Silva et al., 2004, 2006) are used. The corresponding values for the fracture toughness and the strength of the adhesive are taken from standard test results reported in literature. Fig. 3 shows the results of this comparison. It can be seen that the failure load predictions of both models yield approximately the same failure loads. The failure load predictions are in good agreement with the experimental results. Increased overlap lengths lead to increased failure loads but it is well known that this effect gets smaller for larger overlap lengths. Increased adhesive layer thicknesses lead to decreased failure loads although the stresses are less concentrated when the adhesive layer is thicker. Both models render these parameter effects correctly. The finite crack lengths predicted in the model are in the range of $.3 < a < 1.8$mm, which are typical values for predicted finite crack lengths (e.g. Hebel and Becker (2008)). In the numerical analysis the angle of the crack was determined by the solution of the coupled criterion as well. All crack angles ϕ were lower than 4°. In case of the SLJ configurations studied in the comparison fixing the angle of the crack to 0° did not change the results significantly. A similar failure analysis has been developed by Mendoza-Navarro et al. (2013) and Moradi et al. (2013). But in these analyses the effect of non-linearity due to the bending deformation of the adherends was not studied. Consequently these models gave lower failure load predictions at a max of approximately -25% for the presently studied SLJ configurations.

4. Conclusion

In this work weak stress singularities are discussed briefly and the need for advanced criteria to study crack initiation at such stress concentrations is shown. It is shown that Finite Fracture Mechanics, that considers the instantaneous formation of cracks of finite size can be used together with coupled stress and energy criteria to develop a physically sound criterion for crack initiation. The relationship to the Theory of Critical Distances is discussed and it is shown that no empiric length scale is required in the use of coupled criteria but only two fundamental failure parameters — the strength and the toughness — suffice. Two failure models for crack initiation in adhesively bonded single lap joints are presented. A comparison to experimental results shows a good agreement of the failure predictions of both failure models for several joint configurations. The size effect that leads to decreased failure loads with thicker adhesive layer is rendered correctly by both models.

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